Jamming transitions in a mixed traffic cellular automata model

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Abstract

In this work, we focus on the loading of passengers of buses. We use a modified Nagel-Schreckenberg (NaSch) cellular automata model that incorporates buses and passenger pickups and study the dynamics of buses in such a system. In the Nagel-Schreckenberg model, vehicles follow rules for acceleration, deceleration, random slowdown, and movement. Our modifications for buses involve stopping at a cell occupied by passengers for loading. In this model, we allow passengers to spawn along any part of the road. We analyze the interplay between the passenger arrival probability and density of buses in the system and show transitions from platooning to homogenous states with decreasing passenger arrival probability and increasing bus density. Crossover from platooning to homogeneous phases can be induced by decreasing the passenger arrival rates, and increasing the density of buses.

Keywords: Traffic, Cellular automata, Crossover

1 Introduction

Public transportation is universally acknowledged as a fundamental component in solving traffic congestion. Together with rail systems, buses form the backbone of medium- to long-haul modes of people transport. Since the interaction of buses and passengers introduces complex behavior in transportation systems, more so in cities that do not have designated stops, several models have been proposed to study bus traffic [1, 2]. In these models, a delay in the arrival time of a bus leads to more passengers waiting for the bus, which in turn cause further delays the delayed bus. At the same time, succeeding buses will find less passengers waiting for them at stations which allows them to catch up to the delayed bus. Thus, buses form platoons (or bunches). These models also show a transition from the platooning state to a homogeneous state with increasing bus density.

The tendency of buses to form platoons is problematic for public transport. In an ideal scenario, an efficient transport system would try to maintain equal time intervals between arrivals of vehicles. However, an equal headway configuration of public transport vehicles is unstable, thus public transport has a high tendency to be inefficient. Since the instability of an equal headway configuration is inherent to the interaction between public transport vehicles and passengers, approaches to maintain equal headways should consider both traffic and passenger behavior [3].

Bus route models omit the interaction between buses and other vehicle types. As buses (and other forms of public transport) often stop to pick up passengers as compared to private vehicles, one expects these vehicular interactions to play an important role in the dynamics of traffic flow. Even a single bus in two lane traffic can alter traffic states and jamming transition [4].

In this work, we study the transitions induced by vehicle density and passenger arrival rate on a Nagel-Schreckenberg model modified to include buses. We show how the interplay between vehicle density and passenger arrival rates affect the crossover transition from homogeneous to platooning states.

2 Model

Our model draws inspiration from the Nagel Screckenberg (NaSch) [5, 6] model and Bus Route Model (BRM) [1]. There are two components to this hybrid model. The road model $\mathcal{R}$ has a single lane of length L sites, with periodic boundary conditions. A vehicle can interact with passengers (buses), or ignore passengers (cars). A sidewalk model $\mathcal{S}$ with one lane of length L sites is updated in parallel to the road model. Vehicle states are lane l^i, position x^i, and speed $0 \leq v^i \leq v_{\max}$. A site on the road model is occupied by a car if $\mathcal{R}(x) = 1$, and a bus if $\mathcal{R}(x) = 2$. Passengers occupy the sidewalk if $\mathcal{S}(x) = 1$. However as in the BRM, we impose the restriction that $\mathcal{R}(x) = 2$ and $\mathcal{S}(x) = 1$ cannot occur simultaneously (i.e. the corresponding sites of the road and sidewalk cannot have a bus and passengers).

For simplicity, both vehicle types have similar parameters $v_{\max}$ and p_{slow}, with the only difference between the two being the additional interaction with pedestrians for buses. Additionally, we will not be considering the passenger capacity of the bus.

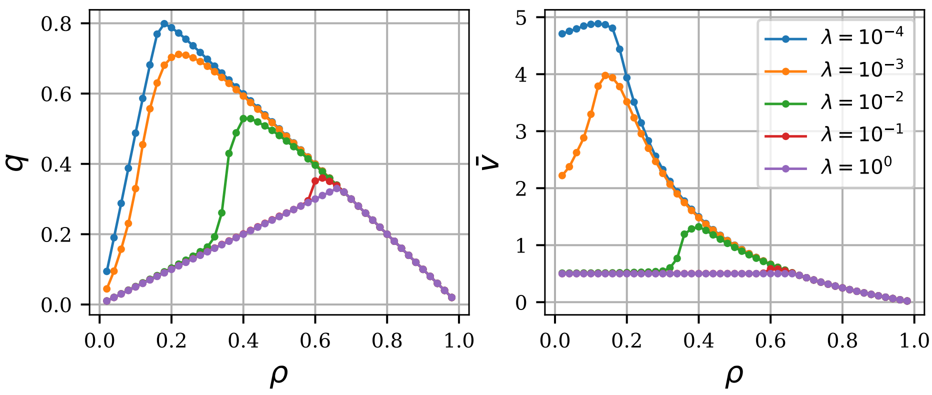

Figure 1: Plots of flow vs. density and speed vs. density for different passenger arrival rates. All vehicles are buses ($f_B = 1$). Jumps in q and $\bar{v}$ are observed at different densities which appear to have a dependence on λ.

Realizations of the model involves assigning vehicle density ρ and passenger arrival rate λ. Vehicle states are updated in random sequential order at each timestep t ($\delta t = 1$) following these rules:
Rule 1: Acceleration: $v^i_{t+1} = \min(v^i_t + 1, v_{\max})$
Rule 2: Bus Loading: $v^i_{t+1} = 0$ if $\mathcal{S}(x^i_t) = 1$; $\mathcal{S}(x^i_t) = 0$
Rule 3: Deceleration: $v^i_{t+1} = \min(v^i_{t+1}, \frac{\Delta x^i}{\delta t})$
Rule 4: Random Slowdown: $v^i_{t+1} = \max(v^i_{t+1} - 1, 0)$ with probability p_{slow}
Rule 5: Forward Movement: $x^i_{t+1} = x^i_t + v^i_{t+1}\delta t$

The headway Δx^i is the number of empty cells ahead of vehicle i, p_{slow} represents fluctuations in driver behavior, such as slowing down to pick up passengers at arbitrary stops, or to find street parking. For regular vehicles, **Bus Loading** is skipped. The evolution of the model over time forces vehicle interactions, which give rise to complex dynamics of traffic flow.

We measure the time averaged flow q of the system, defined as $q = \frac{1}{T}\frac{\sum_t \sum_i v^i_t}{L}$, where v^i_t is the speed of the i^{th} vehicle at time t. The average speed of vehicles in the system is simply $\bar{v} = \frac{q}{\rho}$.

We allow for a transient simulation time of $T_\tau = 1000$ timesteps so as not to include transient behavior in the data [7]. Data measurement was done for $T = 2000$ timesteps for all realizations. Fifty trials were done for each set of parameters ρ, λ, and f_B.

3 Results and Discussion

In this work, we focus on the case where all vehicles are buses. We expect the model to exhibit two different characteristics at the extreme values of λ. For the case $\lambda \to 0$, no passengers arrive at cells. With no passengers to pick up, buses do not slow down, and we recover the NaSch model. In the NaSch model, we expect a phase transition to occur at $\rho_{\text{crit}} = \frac{1}{v_{\max}+1}$, which is the maximum allowable density where vehicles have enough headway to avoid slowing down [5]. For the case of $v_{\max} = 5$, $\rho_{\text{crit}} \approx \frac{1}{6}$. Increasing the density beyond ρ_{crit} induces a phase transition from free-flowing traffic to congestion.

On the opposite extreme end, as $\lambda \to 1$, buses are forced to stop after every other timestep, since it is guaranteed that there will be a passenger waiting for a bus at the next cell. Thus, buses in this system can be seen to move an average speed of $v_{\max} = 0.5$ sites per timestep. A phase transition can be observed to occur at $\rho \approx \frac{2}{3}$, which corresponds to the critical density for $v_{\max} = 0.5$.

Figure 1 illustrates the relationships between flow, speed, and density for passenger arrival rates $0 < \lambda < 1$. There appears to be two competing trends that govern the dynamics of the system. The interaction of buses with passengers tends to drive the average speed of the system to $\bar{v} = 0.5$, and the dynamics of the NaSch model without buses ($\lambda \to 0$ or equivalently, $f_B = 0$). For the values of $\lambda = [10^{-3}, 10^{-2}, 10^{-1}]$, we can observe that for low densities, slow buses picking up passengers is the dominant behavior. However, increasing the density of buses can suddenly shift the dominant behavior of the system towards the dynamics of the regular NaSch model. The location of this sudden shift appears to occur at increasing densities, for increasing values of the passenger arrival rate.

This interplay between λ and ρ suggests that low values of density allow for the accumulation of

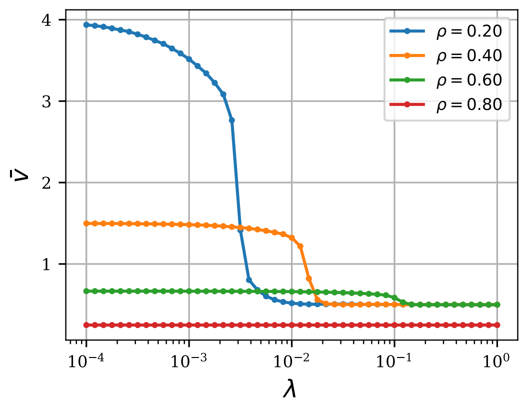

Figure 2: Comparisons of speed as a function of the passenger arrival rate λ for different densities with 25% buses. Sudden drops in vehicle speeds as λ is increased indicate a crossover from homogeneous to platooned flow. However, no transition is observed for high densities ($\rho = 0.8$).

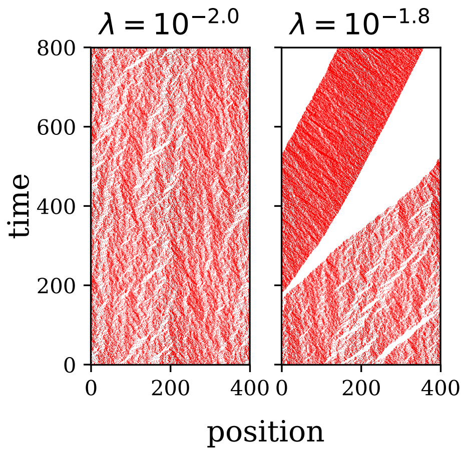

Figure 3: Spatio-temporal diagrams of the system near the transition point of $\rho = 0.4$, for low passenger arrival rate ($\lambda = 10^{-2}$, left) and high passenger arrival rate ($\lambda = 10^{-1.8}$, right). Increasing λ induces a transition from homogeneous to platooned flow.

passengers on different sites of the road, which in turn forces buses to stop more frequently to pick up passengers. On the other hand, increasing the density of buses not only accounts for the pick-ups of these passengers, but also prevents the accumulation of passengers in the system. An interesting consequence of this is that the average speed of the system actually increases.

We can look at the interplay between ρ and λ in another way, by varying λ for fixed values of ρ. Figure 2 shows a crossover from congested traffic to platooning as a function of the passenger arrival rate λ. For a particular density, once λ becomes sufficiently large, passengers fill up the space ahead of buses, such that the leading bus eventually has to stop to pick up passengers at every timestep. This sets the formation of a platoon of vehicles behind the leading bus, while at the same time allowing for more passengers to arrive at empty cells ahead of the platoon of vehicles.

Figure 3 illustrates the crossover from a homogeneous to platooning phase for two different values of λ for the case of $\rho = 0.4$. A platoon develops from an initial homogeneous configuration ($\rho = 0.4$, $\lambda = 10^{-1.8}$). The development of these platoons stem from fluctuations in the speed of buses that pick up passengers. The absence of passengers to pick up causes buses behind the leading bus to clump together. In effect, the formation of this platoon leaves large gaps, which passengers fill up. This increases the waiting time of passengers and slows down the movement of the platoon of buses.

We can make the two observations of our model. First, we see that there exists a $\rho^*(\lambda)$ responsible for a crossover behavior from a platooned phase to a fairly homogeneous phase. When $\rho < \rho^*$, the low density of buses gives more time for passengers to accumulate in the gaps between buses. Buses will have a tendency to clump together, creating larger gaps, which both aid in the formation of the single platoon.

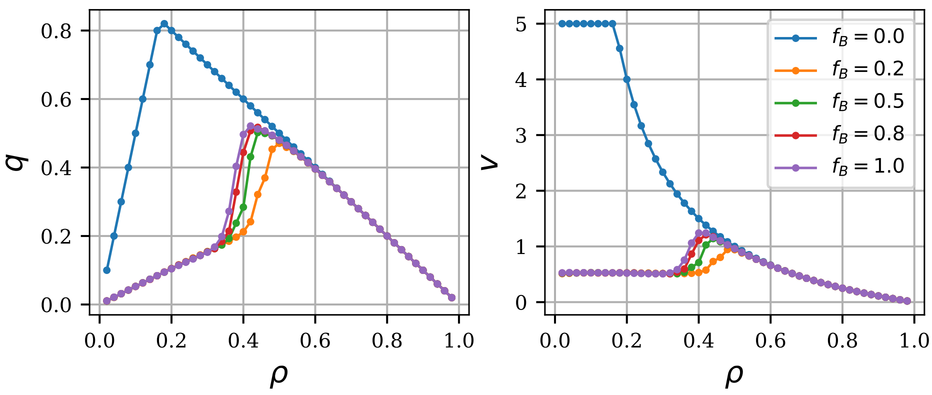

Figure 4: Plots of flow vs. density and speed vs. density for different fraction of buses. The passenger arrival rate is $\lambda = 10^{-2}$. Jumps in q and $\bar{v}$ are observed at higher densities as f_B is increased.

When $\rho > \rho^*$, the gaps between buses are sufficiently small such that passengers have less time to arrive in large numbers and cause a cascade of slowdowns. Thus, smaller or no platoons are formed, and we obtain a homogeneous phase.

Secondly, the crossover from a platooned to homogeneous phase is also determined by $\lambda^*(\rho)$. For arrival rates $\lambda < \lambda^*$, the system would be found in the homogeneous phase, while for $\lambda > \lambda^*$ the system will exhibit platooning. The dependence on ρ in this case is only up to $\rho \approx \frac{2}{3}$, beyond which the density dependent dynamics of the NaSch dominates the system.

We also look at the case of mixed traffic. From Fig. 4, we can see that the transition from a platooning to homogeneous phase occurs at lower densities with increasing f_B. This observation is still consistent with the notion of the transition from platooning to homogeneous phase due to increasing density of buses. In this case, lower f_B also lowers the effective density of buses, which places the transition at a higher density than the case of higher f_B.

4 Summary and Conclusions

We created a hybrid model that combines features found in the Nagel-Schreckenberg and Bus Route Models. In this model, we observed three phases of traffic: homogeneous free flow, homogeneous congestion, and platooning. While we did not find phase transitions in the regime of $0 < \lambda < 1$, we observed crossovers characterized by $\rho^*(\lambda)$ and $\lambda^*(\rho)$. This crossover is distinct from the phase transition described by ρ_{crit}, which marks the transition from homogeneous free flow to homogeneous congested traffic. Our simulations show that decreasing passenger arrival rates, and increasing bus densities induces a crossover from platooning to homogeneous phases. In cases of mixed traffic, we also showed that the crossover behavior depends on the effective density of buses.

Acknowledgments

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